IFMASS10 – Lecture 1

MATHEMATICAL FOUNDATION OF LINEAR ELASTIC MECHANICS OF FRACTURE

Michael P. Wnuk
College of Engineering and Applied Sciences
University of Wisconsin – Milwaukee

ABSTRACT

The first principles of the linear elastic fracture mechanics will be reviewed with special emphasis on the mathematical tools that are applicable and useful also in the nonlinear range of the theory. Linear and nonlinear material response will be described in the context of some well established fracture criteria, applicable to both linear and nonlinear ranges and scale intervals. Material responses to deformation and fracture within macro, meso and nano scale ranges are included in the contents of the lecture. Atomistic models and fractal models of fracture will be compared with the theory that results from Continuum Mechanics. In particular, attention will be focused on the various forms of fracture criteria used to predict initiation of crack growth and the onset of catastrophic fracture. The key topics to be discussed in this lecture are listed below.

1) Energy criterion based on the concept of potential energy of the system (system is understood as a solid weakened by crack); used initially by Griffith as a global approach.
2) Crack driving force criterion introduced by Irwin and based on a concept of crack closure (or opening) occurring at an infinitesimal distance. Concepts of finite K-factors and G-forces employed to mathematically describe singular stress and strain fields near the crack tip.
3) Path independent contour integral of Rice. Fundamental relationships between K-factors, specific fracture energy G, cohesion modulus K_{coh} and J-integral.
4) Crack opening displacement (COD) criterion of Wells applicable to initiation of fracture in ductile materials. Proof of equivalence between J-integral criterion and the COD criterion.
5) Time dependent fracture. Crack initiation and motion as represented by the equations of Schapery, Knauss and Wnuk. Rate-dependent characteristic material resistance curve and its applications in fail-safe design.
6) Final stretch criterion of Wnuk, the so-called “delta-COD” criterion applied to the problem of crack growth initiation (seen as a local instability), the subcritical phase of crack growth and the onset of catastrophic fracture (seen as a global instability).
7) Universal material resistance curve, the R-curve, as predicted by the Wnuk-Rice-Sorensen equation. Applications of R-curve data in design against ductile and/or rate-dependent fracture.